3-6. Two masses $m_1 = 100$ g and $m_2 = 200$ g slide freely in a horizontal frictionless track and are connected by a spring whose force constant is $k = 0.5$ N/m. Find the frequency of oscillatory motion for this system.

3-7. A body of uniform cross-sectional area $A = 1$ cm^2 and of mass density $\rho = 0.8$ g/cm^3 floats in a liquid of density $\rho_0 = 1$ g/cm^3 and at equilibrium displaces a volume $V = 0.8$ cm^3. Show that the period of small oscillations about the equilibrium position is given by

$$\tau = 2\pi\sqrt{V/gA}$$

where g is the gravitational field strength. Determine the value of τ.

3-8. A pendulum is suspended from the cusp of a cycloid* cut in a rigid support (Figure 3-A). The path described by the pendulum bob is cycloidal and is given by

$$x = a(\phi - \sin\phi), \qquad y = a(\cos\phi - 1)$$

where the length of the pendulum is $l = 4a$, and where ϕ is the angle of rotation of the circle generating the cycloid. Show that the oscillations are exactly isochronous with a frequency $\omega_0 = \sqrt{g/l}$, independent of the amplitude.

*The reader unfamiliar with the properties of cycloids should consult a text on analytic geometry.

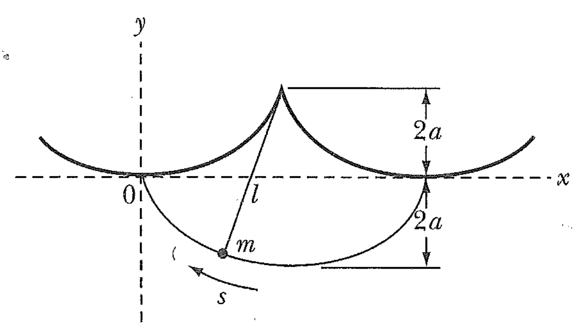

FIGURE 3-A Problem 3-8.

3-9. A particle of mass m is at rest at the end of a spring (force constant $= k$) hanging from a fixed support. At $t = 0$, a constant downward force F is applied to the mass and acts for a time t_0. Show that, after the force is removed, the displacement of the mass from its equilibrium position ($x = x_0$, where x is down) is

$$x - x_0 = \frac{F}{k}[\cos\omega_0(t - t_0) - \cos\omega_0 t]$$

where $\omega_0^2 = k/m$.

5.8 ⋆ **(a)** If a mass $m = 0.2$ kg is tied to one end of a spring whose force constant $k = 80$ N/m and whose other end is held fixed, what are the angular frequency ω, the frequency f, and the period τ of its oscillations? **(b)** If the initial position and velocity are $x_o = 0$ and $v_o = 40$ m/s, what are the constants A and δ in the expression $x(t) = A\cos(\omega t - \delta)$?

5.9 ⋆ The maximum displacement of a mass oscillating about its equilibrium position is 0.2 m, and its maximum speed is 1.2 m/s. What is the period τ of its oscillations?

5.10 ⋆ The force on a mass m at position x on the x axis is $F = -F_o \sinh \alpha x$, where F_o and α are positive constants. Find the potential energy $U(x)$, and give an approximation for $U(x)$ suitable for small oscillations. What is the angular frequency of such oscillations?

5.11 ⋆ You are told that, at the known positions x_1 and x_2, an oscillating mass m has speeds v_1 and v_2. What are the amplitude and the angular frequency of the oscillations?

5.12 ⋆⋆ Consider a simple harmonic oscillator with period τ. Let $\langle f \rangle$ denote the average value of any variable $f(t)$, averaged over one complete cycle:

$$\langle f \rangle = \frac{1}{\tau}\int_0^\tau f(t)\,dt. \tag{5.103}$$

Prove that $\langle T \rangle = \langle U \rangle = \frac{1}{2}E$ where E is the total energy of the oscillator. [*Hint:* Start by proving the more general, and extremely useful, results that $\langle \sin^2(\omega t - \delta)\rangle = \langle \cos^2(\omega t - \delta)\rangle = \frac{1}{2}$. Explain why these two results are almost obvious, then prove them by using trig identities to rewrite $\sin^2\theta$ and $\cos^2\theta$ in terms of $\cos(2\theta)$.]

5.13 ⋆⋆ The potential energy of a one-dimensional mass m at a distance r from the origin is

$$U(r) = U_o\left(\frac{r}{R} + \lambda^2\frac{R}{r}\right)$$

for $0 < r < \infty$, with U_o, R, and λ all positive constants. Find the equilibrium position r_o. Let x be the distance from equilibrium and show that, for small x, the PE has the form $U = \text{const} + \frac{1}{2}kx^2$. What is the angular frequency of small oscillations?